TRANSACTION MANAGEMENT IN SERVICE ORIENTED SYSTEM WITH MULTI CLASS NEGATIVE AND POSITIVE CLIENTS

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Abstract: A new class of queuing networks with "negative and positive" customers was introduced and shown to have a non-standard product form [4]; since then this model has undergone several generalizations [5, 6, 7, 8]. Positive customers are identical to the usual customers of a queuing network, while a negative customer which arrives to a queue simply destroys a positive customer. We call these generalized queuing networks in Networks. In this paper we extend the basic model of [4] to the case of multiple classes of positive customers, and multiple classes of negative customers. As in known multiple class queuing networks a positive customer class is characterized by the routing probabilities and the service rate parameter at each service center while negative customers of different classes may have different "customer destruction" capabilities. In the present paper all service time distributions are exponential and the service centers can be of the following types: FIFO (first-in-first-out), LIFO/PR (last-in-first-out with preemption), PS (processor sharing), with class dependent service rates.

1. INTRODUCTION

In a recent paper [4], a new class of queuing networks in which customers are either "negative" or "positive" was introduced. Positive customers enter a queue and receive service as ordinary queuing network customers, while a negative customer will vanish if it arrives to an empty queue and will reduce by one the number of positive customers in queue otherwise. Negative customers do not receive service. Positive customers which leave a queue to enter another one can become negative or remain positive. It has been shown [4] that networks of queues with a single class of positive and negative customers have a product form solution if the external positive or negative customer arrivals are Poisson, the service times of positive customers are exponential and independent, and if the movement of customers between queues is Markovian.

We shall call these generalized queuing networks "Networks" in order to distinguish them from the usual queuing network models. The single server queue with negative and positive customers has been examined in [5]. Stability conditions for these networks have been discussed in [6], while "triggers" which are specific customers which can order the rerouting of customers, and batch removal of customers by negative customers, have been introduced in [7,8].

Networks can be used to represent different systems. The initial model in [4] was motivated by the analogy with neural networks [3]: each queue represents a neuron, and customers represent excitation (positive) or inhibition (negative) signals. Another possible application is to multiple resource systems: positive customers can be considered to be resource requests, while negative customers can correspond to decisions to cancel such requests.

In this paper we investigate networks with multiple classes of positive customers and one or more class of negative customers.

We consider networks with three types of service centers which correspond to the following service disciplines:

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- Type 1 : first-in-first-out (FIFO),
- Type 2 : processor sharing (PS),
- Type 4: last-in-first-out with preemptive resume priority (LIFO/PR).

Here we are using the standard nomenclature of the BCMP theorem [l]. Note that we exclude from our model the Type 3 service centers with an infinite number of servers which are not covered by our results. In our model, the number of negative customer classes allowed will depend on the type of service center considered.

In Section 2 we prove that these multiple class Networks, with Type E, 2 and 4 service centers, have product form.

It was shown in [4] that the customer flow equations of this type of network are non-linear, so that the existence of their solutions is not an obvious matter except for feed-forward networks. Thus the network stability conditions and the existence of the solutions to customer flow equations cannot be handled in a routine manner as with BCMP networks [l]. The stability conditions (i.e. necessary and sufficient conditions for the existence of a solution) for such networks have been solved recently in [6], and the approach is applied to the present model in Section 4.

2 The model

We consider networks with an arbitrary number *N* of queues, an arbitrary number of positive customer classes K, and an arbitrary number of negative CUB toner classes S. As in [4] we are only interested in open Networks. Indeed, if the system is closed, then the total number of customers will decrease as long as there are negative customers in the network.

External arrival streams to the network are independent Poisson processes concerning positive customers of some class *k* or negative customers of some class c. We denote by $A_{i,k}$ the external arrival rate of positive customers of class *k* to queue *i* and by $\lambda_{i,m}$ be the external arrival rate of *negative* customers of class m to queue *i.*

Only positive customers are served, and after service they may change class, service center and nature (positive to negative), or depart from the system. The movement of customers between queues, classes and nature (positive to negative) is represented by a Markov chain.

At its arrival in a non empty queue, a negative customer selects a positive customer in the queue in accordance with the service discipline at this station. If the queue is empty, then the negative customer simply disappears. Once the target is selected, the negative customer tries to destroy the selected customer. A negative customer of some class m succeed to destroy the selected positive customer of some class *k* at service center *i* with probability $K_{i,m,k}$. And with probability (1 - $K_{i,m,k}$) it does not succeed. A negative customer disappears as soon as it tries to destroy its targeted customer. Recall that a negative customer is either exogenous, or is obtained by the transformation of a positive customer as it leaves a queue.

A positive customer of class *k* which leaves queue *I* (after finishing service) goes to queue *j* as a positive customer of class *1* with probability *P+[i,j] [k,* 13, or as a negative customer of class m with probability *P-[i, j] [k, m].* It may also depart from the network with probability d[i, *k].*

We shall assume that *P+[i,i][k,Z]* = 0 and *P-[i,i][k,m]*= 0 for all *i , k , l ,m*s o that customers are not allowed to return to the queue they have just left. Though this assumption is without loss of generality for positive customers, it is indeed a restriction of the model for negative customers since it avoids double simultaneous customer “departures” at a queue.

Obviously we have for all *i, k*

$$\sum_{j=1}^{N}\sum_{l=1}^{R} P^{+}[i,j][k,l] + \sum_{j=1}^{N}\sum_{m=1}^{S} P^{-}[i,j][k,m] + d[i,k] = 1 \quad (1)$$

We assume that all service centers have exponential service time distributions. In the three types of service centers, each class of positive customers may have a distinct service rate $\mu_{i,k}$ However, when the service center is of Type 1 (FIFO) the following quantity, which takes into account the service rate, must be a constant for all class *k* of positive customer.

$$\mu_{i,k} + \sum_{m=1}^{S} K_{i,m,k}\lambda_{i,m} = c_i \quad (2)$$

The following constraints on the deletion discipline and deletion probability are assumed to exist.

- For a Type 1 server, an arriving negative Customer selects the one being served; and the following constraint must hold for all stations *i* of type 1 and classes of negative customers m such that $\sum_{j=1}^{N} \sum_{i=1}^{R} P-[j,i][l,\mathrm{m},]>0$:

$$K_{i,m,k} = K_{i,m,p} \quad (3)$$

For all classes of positive customers k and p, and any negative customer class m. This constraint implies that a negative customer of some class m arriving from the network does not "distinguish" between the positive customer classes it will try to delete, and that it will treat them all in the same manner.

- For a Type 2 server, the probability that any one positive customer of the queue is selected by the arriving negative customer is *1/c* if *c* is the total number of customers in queue.

- In a Type 4 server, the only restriction is that the positive customer selected by a negative customer is the one in service.

For Type *1* service centers, one may consider the following conditions which are simpler than (2) and (3):

$$\begin{aligned} \mu_{ik} &= \mu_{ip} \\ K_{i,m,k} &= K_{i,m,p} \end{aligned} \quad (4)$$

For all classes of positive customers k and p, and all classes of negative customers m. Note however that these new conditions are more restrictive, though they do imply that (2, 3) hold.

2.1 State Representation

- We shall denote the state at time t of the queuing network by the vector *z(t)= (z l (t) ,. .., ZN(t))* . Here *zj(t)* represents the state of service center i. The veG tor t = *(2 1 , ..., X N)* will denote a particular value of the state and *l z i l* will be the total number of customers in queue *i* for state t.

For Type *1* and Type 4 servers, the instantaneous value of the state x_j of server *i* is represented by the vector $(r_{j,j})$ whose length is the number of customers in the queue and whose *jth* element is the class index of the *jth* customer in the queue. Furthermore, the customers are ordered according to the service order (FIFO or LIFO); it is always the customer at the head of the list which is in service. We denote by $r_{i,l}$ the class number of the customer in service and by $r_{j,\infty}$ the class number of the last customer in the queue.

For a PS (Type 2) service station, the instantaneous value of the state *t i* is represented by the vector $(x_{j,k})$ which is the number of customers of class *k* in queue *i.*

3 Main Results

Let n(z) denote the stationary probability distribution of the state of the network, if it exists. The following result establishes the product form solution of the network being considered.

Theorem 1 ***Consider a Network with the restrictions indicated above. If the system of non-linear equations:***

$$q_{i,k} = \frac{\Lambda_{i,k} + \Lambda_{i,k}^{+}}{\mu_{i,k} + \sum_{m=1}^{S} K_{i,m,k}[\lambda_{i,m} + \lambda_{i,m}^{-}]} \quad (5)$$

$$\Lambda_{i,k}^{+} = \sum_{j=1}^{N}\sum_{l=1}^{R} P^{+}[j,i][l,k]\mu_{j,l}q_{j,l} \quad (6)$$

$$\lambda_{i,m}^{-} = \sum_{j=1}^{N}\sum_{l=1}^{R} P^{-}[j,i][l,m]\mu_{j,l}q_{j,l} \quad (7)$$

Has a solution such that for each pair i, k:

0 < $q_{i,k}$ *and* for *each Station i* : $\sum_{k=1}^{R} q_{i,k} < 1$

Then the stationary distribution of the network state is

$$\Pi(x) = G \prod_{i=1}^{N} g_i(x_i) \quad (8)$$

Where each $g_i(x_i)$ depends on the Type of service center i. The $g_i(x_i)$ in (5) have the following form :

FIFO If *the service center is of Type 1, then*

$$g_i(x_i) = \prod_{n=1}^{|x_i|} [q_{i,r_{i,n}}] \tag{9}$$

PS *If the service center is of Type 2, then*

$$g_i(x_i) = |x_i|! \prod_{k=1}^{R} \frac{(q_{i,k})^{x_{i,k}}}{x_{i,k}!} \tag{10}$$

LIFO/PR *If the service center is of Type 4, then*

$$g_i(x_i) = \prod_{n=1}^{|x_i|} [q_{i,r_{i,n}}] \tag{11}$$

and G is the normalization constant.

The proof, which we do not give here, is based on simple algebraic manipulations of global balance equations, since it is not possible to use the "local balance" equations for customer classes at stations because of the effect of negative customer arrivals.

As in the BCMP *[l]* theorem, we can also compute the steady state distribution of the number of customers of each class in each queue. Let *yi* be the vector whose elements are *(y i , k)* the number of customers of class *k* in station *i*. Let *y* be the vector of vectors *(P i)* .

Theorem 2 If the system of equations (5), (6) and (7) has a solution then, the steady state distribution n(y) is given by

$$\pi(y) = \prod_{i=1}^{N} h_i(y_i) \tag{12}$$

where the marginal probabilities h i (y i) have the following form :

$$h_i(y_i) = (1 - \sum_{k=1}^{R} q_{i,k})|y_i|! \prod_{k=1}^{R} [(q_{i,k})^{y_{i,k}}/y_{i,k}!] \tag{13}$$

4 Existence of the Solution to the Traffic Equations

Unlike BCMP or Jackson networks [l], the customer flow equations (5), (6) and (7) of the model we consider are non-linear. Therefore issues of existence and uniqueness of their solutions have to be examined.

In particular, our key result depends on the *exis*tence of solutions to (5), (6), (7). Thus the issue of existence and uniqueness of solutions to these traffic equations is central to our work.

Define the following vectors:

Λ^+ with elements $\Lambda^+_{i,k}$
λ^- with elements $\lambda^-_{i,k}$
Λ with elements $\Lambda_{i,k}$ and
λ with elements $\lambda_{i,k}$

Furthermore, denote by *P*+ the matrix of elements *{P+[i,j] }*, and by *P*- the matrix whose elements are *{P-[i,j] }*.

Let *F* be a diagonal matrix with elements 0 *2 F i , k5* 1. Equations (6) and (7) inspire us to write the following equation: {P-[i,jl}.

$$\Lambda^+ = \Lambda^+ F P^+ + \Lambda, \qquad \lambda^- = \Lambda^+ F P^- + \lambda \tag{14}$$

or, denoting the identity matrix I, as

$$\Lambda^+(\mathbf{I} - FP^+) = \Lambda, \tag{15}$$

$$\lambda^- = \Lambda^+ F P^- + \lambda. \tag{16}$$

Proposition 1: If *P*+ is a sub stochastic matrix which does not contain erotic classes, then equations (15) and (16) have a solution (A+, A-).

PROOFT: he series *C,",.,(FP+)"* is geometrically convergent, since F *5* I, and because – by assumption - *P*+ is sub stochastic and does not contain any erotic classes (see Kemeny and Snell [6] pp 43 q). Therefore we can write (15) as

$$\Lambda^+ = \Lambda \sum_{n=0}^{\infty} (FP^+)^n, \tag{17}$$

so that (16) becomes

$$\lambda^- - \lambda = \Lambda \sum_{n=0}^{\infty} (FP^+)^n FP^-. \quad (18)$$

Now denote z = $\boldsymbol{\lambda}^- - \boldsymbol{\lambda}$, and call the vector function

$$G(z) = \Lambda \sum_{n=0}^{\infty} (FP^+)^n FP^-$$

Note that the dependency of G on *l* comes from *F,* which depends on $\boldsymbol{\lambda}^-$.

It can be seen that G : [O,G(O)] --+ [O,G(O)] and that it is continuous. Therefore, by Brower's fixed-point theorem

$$z = G(z) \quad (19)$$

has a fixed point *l**. This fixed point will yield the solution of (15) and (16) as:

$$\lambda^-(y^*) = \lambda + y^*, \qquad \Lambda^+(z^*) = \Lambda \sum_{n=0}^{\infty} (F(z^*)P^+)^n, \quad (20)$$

completing the proof of Proposition 1.

Proposition 2 Equations (6), (7) have a solution.

PROOF: This is result is a direct consequence of Proposition 1, since we can see that (5), (6) and (7) are a special instance of (21). Indeed, it suffices to set

$$F_{i,k} = \frac{\mu_{i,k}}{\mu_{i,k} + \sum_{m=1}^{S} K_{i,m,k}[\lambda_{i,m} + \lambda_{i,m}^-]} \quad (21)$$

and to notice that $0 \leq F_{i,k} \leq 1$, and that (6), (7) now have taken the form of the generalized traffic equations (21). This completes the proof of Proposition 2.

The above two propositions state that the traffic equations *always* have a solution. Of course, the product form (8) will only exist if the resulting network is stable. The stability condition is summarized below.

Theorem 3 Let z* be a solution of z = G(z) obtained by setting F as in (27). Let $\lambda^-(z^*)$, $\Lambda^+(z^*)$ be the corresponding traffic values, and let the qi,k(z*) be obtained from (5) as a consequence. Then the Gnetwork is stable if all of the O leq $q_{i,k}(z^*) < 1$ for all i, k. Otherwise it is unstable.

5 Conclusions

In this paper we have considered networks of queues with multiple classes of positive customers and multiple classes of negative customers. We have shown that this new type of networks has product form when all service centers are of BCMP type, except for the infinite server case which we do not cover, with class dependent exponential service time distributions (except at FIFO service centers).

Further extensions of these results to more complex service distributions or more complex interactions between positive and negative customers are being considered.

References

1. Baskett F., Chandy K., Muntz R.R., Palacios F.G., Open, closed and mixed networks of queues with diflerent classes of customers, Journal ACM, Vol. 22, No 2, p248-260, April 1975.

2. Fourneau J.M., Computing the steady-state distribution of networks with positive and negative customers, LRI Report, 13th IMACS World Congress on Computation and Applied Mathematics, Dublin, 1991.

3. Gelenbe E., Random neural Networks with Negative and Positive Signals and Product Form Solution, Neural Computation, V1, N4, pp 502-510,1990.

4. Gelenbe E., Product form queuing networks with negative and positive customers, Journal of Applied Probability, Vol. 28, pp 656-663, 1991.

5. Gelenbe E., Glynn P., Sigmann K, Queues with negative customers, Journal of Applied Probability, Vol. 28, pp 245-250, 1991.

6. Gelenbe E., Schassberger R., Stability of Networks, Probability in the Engineering and Informational Sciences, Vol. 6, pp 271- 276, 1992.

7. Gelenbe E., Networks with triggered customer movement, Journal of Applied Probability, 1993 to appear.

8. Gelenbe E., Networks with signals and batch removal, Probability in the Engineering and Informational Sciences, Vol. 7, pp 335- 342, 1993.